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## ABSTRACT

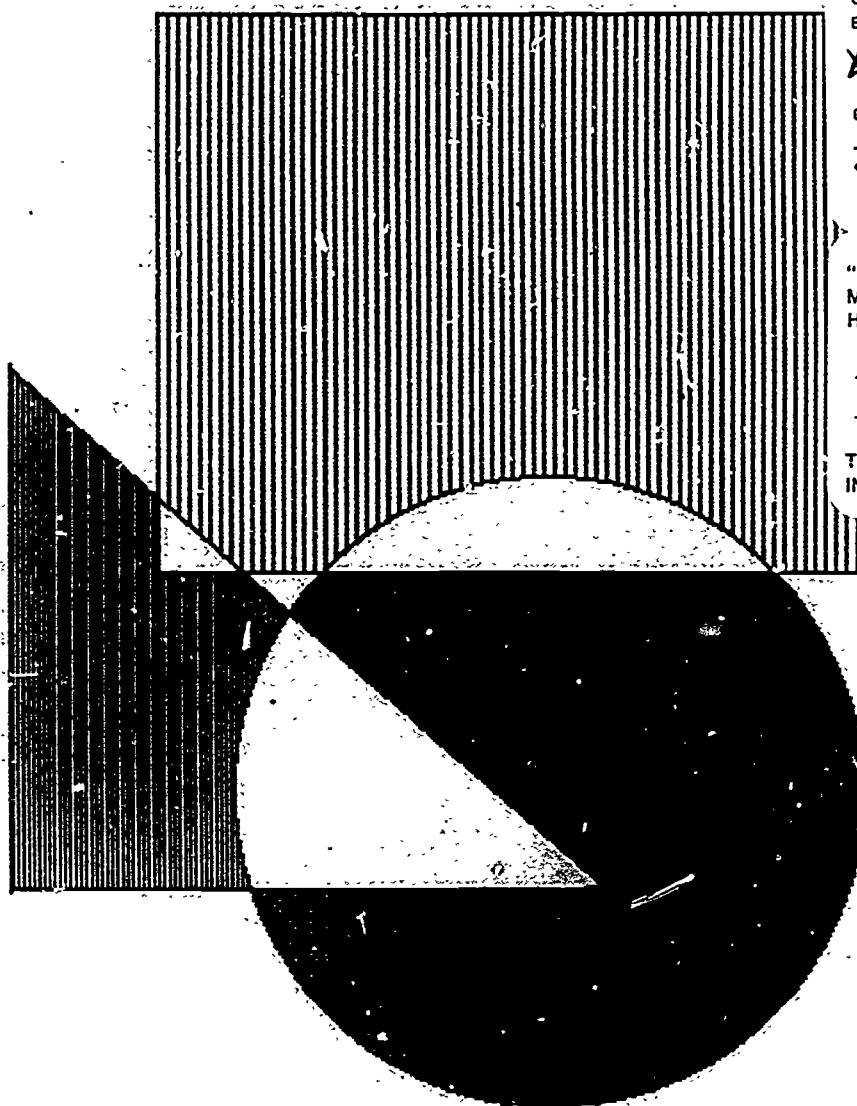
This handbook, and the VITAL Mathematics Series videotapes, contain 12 lessons that model effective use of active learning strategies in elementary mathematics instruction. The lessons are designed for preservice and inservice teacher development programs through courses or individual study. The videotapes that are available separately demonstrate the use of a variety of instructional materials and present alternative means to achieve teaching objectives. Each lesson has the following organization: (1) introduction to the activity; (2) background to the lesson; (3) questions to guide the study of the lesson; (4) further suggestions; (5) materials needed for the lesson; and (6) reference materials.  
(RH)

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VITAL SERIES

# VIDEOTAPES FOR INSERVICE TRAINING FOR ACTIVE LEARNING



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## MATHEMATICS

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**VIDEO TAPES FOR INSERVICE TRAINING FOR ACTIVE LEARNING  
MATHEMATICS**

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VITAL: MATHEMATICS SERIES

Sensitivity to the Nature of human learning reveals that learners must have the opportunity to bring some order and structure to mathematics themselves before they are confronted with the more perfect orderliness developed by others.

Lloyd F. Scott

The last three decades have revealed a wide range of changes in the elementary school mathematics curriculum. The 'New Math' of the 1960's has given way to the 'Back to Basics' math of the 1980's. Recommendations for course content and method abound, often coming from the most influential sources. Yet Fey noted, "the most discouraging feature of three NSF studies is the consistent pattern of great differences between apparent reality of mathematics education in most schools and the recommendations or practices of many prominent teachers, supervisors, and professional organizations. For instance, the studies suggest that a large majority of elementary teachers believe that their sole responsibility in mathematics teaching is to develop student facility in arithmetic computation - this at a time when availability of calculators has made such goals widely questioned." (1982, pg. 41).

Melton noted, "with the emphasis of mathematics as an integral part of the back-to-basics movement, the allocated time for mathematics instruction, especially at the elementary level, has increased. For the most part, however, little attention has been given to the idea of 'time on task'. The actual amount of time that a student is engaged in learning activities is the critical factor, not the allocated time. The issue is: have teachers been provided with the professional development and support necessary to effectively and efficiently utilize the time allocated? (1982, pg. 125).

Jackson expands on this. "Approximately 20 percent of the elementary school day is allocated to mathematics, with the number of minutes increasing as the grade level advances. Approximately 70 percent of teacher time is spent on classroom management not on actual instruction. The implication - how time is spent on mathematics is far more significant than the amount of time allocated. Student performance is higher when more than half of the classroom time is spent on developmental activities in mathematics." (1982, pg. 138).

In light of these statements, it would be helpful to know what kind of experience would help teachers improve their mathematics instruction. Gibney and Karns found that "most mathematics teachers believe they do not need help in lesson planning, teaching lessons, or discipline but they do request help in obtaining information about instructional materials, learning new teaching methods, and using hands-on manipulative materials."

Observations such as these, and a concern that mathematics instruction at the elementary level could be improved, served as stimuli for the production of this manual and the accompanying video tapes.

The project was guided by the following purposes.

1. To promote teacher inservice activities that would stimulate discussion and lead to improved mathematics teaching through use of manipulatives at the elementary level.
2. To provide resources for elementary school teachers to improve their mathematics instruction using active learning strategies.
3. To develop higher order thinking skills and problem solving in mathematics among elementary school students.

The importance of using manipulatives in mathematics instruction cannot be over emphasized. Students need experiences manipulating concrete objects prior to attempting to formulate abstract mathematical ideas. Children should be moved gradually from the concrete to the abstract and be provided with as many different sensory experiences as possible. These opportunities allow student discovery of relationships and formulas and aid students in forming generalizations which may lead to a deeper understanding of concepts.

#### INTRODUCTION TO THE MATHEMATICS TAPES AND LESSONS

Twelve video lessons have been prepared along with this manual to guide their use. The lessons model effective use of active learning strategies in elementary mathematics instruction. They DO NOT represent all ways of using active learning strategies to teach specific ideas, skills, and concepts. They DO demonstrate the use of a variety of instructional materials and present alternative means to achieve teaching objectives. The lessons were taught by teachers from west central Minnesota schools in an unrehearsed setting.

The ideas presented in these materials should be both adopted and adapted to meet unique needs of each particular school setting, grade level and course topic to best enhance the opportunity for learning by elementary students. The twelve lessons include use of the following:

- LESSON 1 Cuisenaire rods with grades 2-3
- LESSONS 2 & 9 Base-ten blocks with grades 2-4 and 5-6
- LESSONS 3 & 4 Geoboard with grades 5-6 and 3-4
- LESSON 5 Tangrams with grades 5-6
- LESSON 6 Extending the mathematics worksheet with grades 2-3
- LESSON 7 Probability with grades 3-4
- LESSON 8 Problem solving skills with grades 5-6
- LESSON 10 Attribute blocks with kindergarten
- LESSONS 11 & 12 Pattern blocks with grades k-1

Each demonstration lesson is from ten to fifteen minutes in length. The lessons are designed for use in preservice and inservice teacher development programs through group based, or individual study. This manual offers teachers and pre/in-service leaders a guide for each lesson. A bibliography of resources and materials is included at the end of this manual.

## **Notes to the workshop leader on use of the tapes**

The video tapes and this instructional guide may be used in a variety of ways. Teachers, like students, should be actively involved in the learning process. Hands-on instruction helps adults as well as young people learn new concepts, skills, and to develop attitudes. As a result, we encourage teachers and preservice and inservice teacher leaders to participate in activities, manipulate instructional materials, and experience the process of learning through use of concrete, hands-on experience followed by in-depth discussion. How this is structured will vary with the instructional setting, time available, and expertise of the teachers and teacher trainers.

The tapes are intended to be used as a part of a workshop in which participants have extensive hands-on experience with the concepts or materials presented in the videotaped lessons. Prepare to view each tape by reading the relevant section of this manual, considering the introduction and questions given for each tape, and raising your own questions for consideration. The tapes may be effectively introduced by simply asking "Here is a \_\_\_\_\_ grade class learning about\_\_\_\_\_. What good teaching practices do you notice? What suggestions would you make to this teacher for improving the lesson?"

### **Ways of using these materials**

1) SELF INSTRUCTION. An individual may select one or more lessons, read the guide materials in this booklet, and watch the video lesson(s). This approach works well if the teacher understands and is familiar with using manipulatives in instruction; is seeking to see how another teacher approaches teaching a concept he/she has already taught; or wants to better understand student responses to using manipulatives and the effect of manipulatives on student learning. Before using these materials and strategies with students it is strongly recommended that teacher's try using the materials themselves to better understand where children may have difficulty with the activity or the instructions.

2) GROUP SELF INSTRUCTION. In this approach, these materials are used to stimulate discussion about instruction among a group of pre or inservice teachers. As with SELF INSTRUCTION, teachers select appropriate lessons, read guide materials, and watch the video lessons. In this case a group of teachers, working together, take time to share their opinions on the demonstration lesson and share their experiences and feelings about using these materials and strategies in their own instruction. This can be a very powerful and effective learning process. Teachers have much to share and these materials may stimulate sharing in which all participants learn from each other and improve instruction for their students.

3) PRESERVICE or INSERVICE TRAINING. In this approach, a teacher leader prepares an instructional lesson for an audience of preservice or inservice teachers. This lesson should include a sequence of activities in which participants do the lesson themselves, view the video lesson, discuss their reactions, and offer suggestions as to how they will/could use the demonstrated

strategy with their students. Brainstorming other ways to use manipulatives to teach or reinforce concept learning is effective for stimulating further creative approaches to instruction. Alternative sequences of activities can be used such as, but not limited by, the following:

- 1) Teach the lesson to the participating pre and/or inservice teachers as if they were elementary school children participating in the lesson (this is a very effective strategy for helping teachers develop a clear understanding of the intent and effect of the lesson on learners). Following a brief discussion of the lesson, show the relevant video lesson and lead a discussion in which the teacher's own experiences with the activity are compared with those of the teacher and students in the video lesson. Conclude by summarizing key points about teaching the lesson and preparing participants to teach the lesson with their own classes.
- 2) Begin the lesson with pre and/or inservice teachers by first showing the video tape. Present the group with a brief introduction to the video lesson and one or two questions to focus their observation and thinking. Either show the whole lesson (10-15 minutes) followed by discussion using the guide questions to start interaction - or - stop the lesson at predetermined places to point out relevant actions, focus attention, and raise further questions for consideration. This process should be followed by actually teaching the lesson - or a related one using the same or similar materials and instructional strategies that were presented in the video lesson. This is followed by discussion to determine reactions, allow time for offering additional suggestions to achieve desired outcomes, and ensure that procedures are understood.

Following either of the above approaches, teachers should be given the opportunity to try teaching part or all of a lesson to their peers in order to develop expertise and comfort with the teaching strategies and materials. This can follow lessons taught by pre or inservice teacher trainers or be done by teachers with their own peers after viewing the video lesson on their own and preparing a sample lesson for use with colleagues.

In all cases, it is important that teachers actively participate in these - or similar - activities in order to develop thorough understanding and facility with the goals and procedures presented in these demonstrations. We hope you find these materials of value and would be pleased if you would take the time to share your experiences with us in order that we can increase our ability to encourage improved mathematics and science instruction for young people.

**LESSON 1**  
**Using Cuisenaire Rods to Introduce Multiplication**

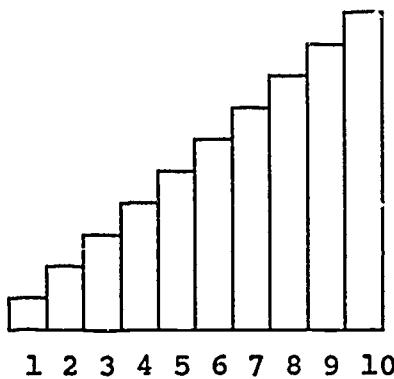
**Teacher: Francie Turner      Demonstration Grade: 2-3**

**Introduction to Cuisenaire Rods:**

Manipulatives allow students to concretely experience mathematical thinking through touch, so that they can make deductions and learn concepts. It is important that students encounter tasks that look different but use the same concept so that mathematical symbols take on personal meaning. Cuisenaire rods can be an effective tool for this purpose.

It is essential for the teacher and students to be both familiar and comfortable with the rods before using them to learn mathematical concepts. It is important to remember that:

1. Each rod color represents a number.
2. When a rod is placed on centimeter graph paper, the numerical value of the rod may be determined by the number of blocks the rod covers.
3. The value of each color is constant, e.g., all reds = 2, all oranges = 10.
4. A rod ladder may be used to show ascending or descending value of the rods.



In the classroom a set of rods can be easily shared by two pupils. Since they may also be effectively used at all elementary levels and easily shared among teachers, they are affordable for any school district.

Using Cuisenaire rods to teach multiplication or division is made easier by following the student worksheets in, Multiplication and Division With Rod Patterns and Graph Paper. The worksheets progress step-by-step and create the foundation for teaching multiplication and division. Cuisenaire rods may also be used to teach or reinforce other mathematical concepts.

### **Background to the Lesson:**

In this lesson the teacher works with her students to make trains of one color with the cuisenaire rods. During the lesson she reinforces the correct written symbolization and meaning of a multiplication fact, the commutative property of multiplication, and grouping of facts into a family of facts.

### **Questions to Guide Study of the Lesson:**

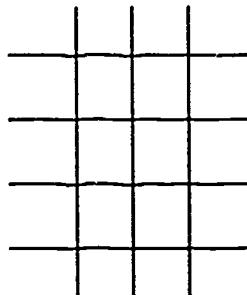
1. What kind of experience should children have with the rods before they are used to teach concepts?
2. What effect will it have on students' responses if, while manipulating the rods, the teacher asks questions or points out ideas to the students?
3. On the worksheet where the children matched the fact with the one-color train, why is the first factor the number of rods and the second factor how much each rod covers? Couldn't these be turned around so that the first factor would tell how much each rod covers and the second factor would tell the number of rods used?
4. Much of the lesson is spent having students tell what they are learning. Why doesn't the teacher simply tell the students that multiplication is repeated addition or that factors may be turned around without changing the product? What are the advantages/disadvantages of this method?

### **Further Suggestions:**

1. Rods placed vertically to form a rectangle may be used to represent multiplication facts. For example:

dark green		
red	red	red

The dark green rod can be matched with 3 red rods.



Make and draw a rectangle for the 3 rods.  
The dimensions are  
\_\_\_\_\_ x \_\_\_\_\_

2. Multiplication flash cards may be made as follows:

- a. Cut up centimeter graph paper to make cards. Each set should be cut to a different size to accommodate the number of squares needed for the set. Multiplication facts of 2 need to have at least 2 centimeter blocks vertically and 9 blocks horizontally. The multiplication facts of 5 need to have at least 5 blocks vertically and 9 blocks horizontally. For example:

$$2 = \\ \times \boxed{\phantom{0} \phantom{0}} \\ 2$$

$$9 = \\ \times \boxed{\phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0}}$$

- b. Students can help make these cards by using magic markers to make the number fact card while the teacher shows the rectangular representation and number fact. The answer to the fact is written on the back of the card in pencil since the magic marker may be seen from the front.
- 3. The concept of repeated addition of the same number may be taught using Base ten blocks (See lesson 2) and Touch Math materials.

**Materials Needed for the Lesson:**

The following materials may be purchased from Cuisenaire Company of America, Inc., 12 Church Street, Box D, New Rochelle, New York, 10805.

- 1. Centimeter Graph Paper - especially designed for use with cuisenaire rods. Sheets may be copied and run off for the students.
- 2. Overhead Projector Rods - a set of colored plastic rods to be used by the teacher while students use their rods at their desks.
- 3. Student Activity Cards - 130 cards divided into ten sets for use in kindergarten through sixth grade. Sets include: Introduction to Rods, Matching Rods, Equalities and Inequalities, Stories about Rods, Geometric Investigations, Patterns with Rods, Measuring Lengths, Multiples and Multiplication Tables, Fractions, and Projects with Rods.
- 4. Texts

Using the Cuisenaire Rods: A text guide for teachers.

Multiplication and Division with Rod Patterns and Graph Paper: 64 pages of student worksheets.

Spatial Problem Solving with Cuisenaire Rods

Idea Book for Cuisenaire Rods at the Primary Level: 164 pages, preschool through second grade.

Hidden Rods/Hidden Numbers: 78 pages, 150 logic problems with clues provided.

Everything's Coming Up Fractions with Cuisenaire Rods: 64 pages, for 3-6th grades.

Touch Math materials (worksheets, posters and desk cards) can also be used to teach these operations. These are available from: Touch Math, P.O. Box 7402, Colorado Springs, Colorado 80 33.

**Notes:**

**LESSON 2**  
**Base Ten Blocks: Visualizing Regrouping**

**Teacher: Francie Turner      Demonstration Grade: 2-4**

**Introduction to Base Ten Blocks:**

One of the major reasons for using base ten blocks is that they allow accurate visual representation of base ten numerals. They use different physical shapes and sizes to show units, tens, hundreds, etc. The student uses these sizes to determine the meaning of place value.

A number of hands-on materials that show numeration can be purchased. For example, number boards use color to show the difference between units, tens, hundreds, etc. They do not help children to visualize that one unit looks different than one ten or one hundred. Commercially made base ten blocks are relatively expensive, but only one kit is needed for illustration by the teacher, in a learning center, or in a one-to-one learning situation.

Individual student materials can be easily made. A 9" X 11" sheet of tagboard is divided vertically into three columns with the headings from left to right: flats, longs and units. A visual representation of each heading is cut out and placed at the top of each column. This tagboard block should be laminated so it will last longer.

Nine by eleven inch sheets of centimeter graph paper are then laminated and cut into blocks, longs and units. (More blocks or hundreds can be made if the centimeter paper is butted together to form larger sheets.) Each student needs one block: 9 flats (100's), 18 longs (10's) and 20 units (1's).

All these may be stored in pint-size zip-lock bags. Note: A block equals 1000. In order to use centimeter paper to represent it, the block would be 10 squares by 100 squares - an impractical size. Hopefully as students learn the concept of hundreds, tens, and units, that knowledge will transfer to the larger place values.

**Background to the Lesson:**

The teacher begins this lesson by reviewing the meaning of the blocks - flats, longs, and units, how to arrange the blocks in columns, and how to use the bank (left over blocks). Students then represent numbers using the blocks. Finally, the teacher guides the class into understanding subtraction using the blocks.

Prior to teaching regrouping using base ten blocks, students need experience with the blocks.

1. They must have time to become familiar with the blocks through free play.
2. They must know the terminology: flats = 100, longs = 10, and units = 1.

3. They must know that only flats can go under the flat column, longs under the long column and units under the units column.
4. They must be able to both use the blocks to represent a number, and know what number is represented by the blocks.
5. They must know how to use a "bank". A bank is all the kit pieces of flats, longs, and units which are not being used. The students must learn that only nine of any shape can be in any column. If there are ten or more, the student has to take those blocks to the bank to trade them in. The process of trading is encountered when problems in addition have too many objects in a column, or in subtraction when more objects are needed in a column.

**Questions to Guide Study of the Lesson:**

1. Why should the teacher encourage students to discuss what they are learning and what they have done?
2. What advantages are there for using numbers within a story to help children solve the problem they are working on?
3. When eliciting student responses to, "what is subtraction?", one student answered, "It's shorter". What was the student trying to say? How might the teacher have handled his/her response?
4. What advantages are there for teaching the four operations using base ten blocks as compared to rote learning steps in the operations?
5. For what type of math student are Base Ten Blocks appropriate? Above average? Average? Below Average? Explain.
6. Is there an upper age limit where the blocks are no longer appropriate?
7. Base Ten Blocks may be used to help students understand the four arithmetic operations. For what other mathematical concepts could the blocks be used?
8. The school district expects teachers to cover the material in the mathematics textbook. How can a teacher justify the time spent using manipulatives?

**Further Suggestions:**

1. Move ahead slowly. Students should be able to handle the materials knowledgeably before moving on to the next step in the learning process. The teacher should encourage children to analyze the meaning of what they are doing during the manipulation stage. Questions such as the following may be helpful:
  - a. What do the blocks mean (when representing a specific number)?

- b. What operation did you choose? Why? What happened?
- 2. Move from the concrete to the abstract gradually. The student should:
  - a. Only use blocks to show a problem that the teacher has given.
  - b. Make up problems, tell the class, and have the class work out the answer.
  - c. 'Walk through' each step of the problem with the blocks while the teacher simultaneously relates the step to the written problem on either the board or the overhead projector.
  - d. Use only numerical symbols with paper and pencil, reverting back to the base ten blocks when mistakes are encountered.
- 3. When adding, have students place the blocks for the second numeral near the bottom of the work board. (Many times they tend to include the numbers already on the board with the numbers they are adding.) Then have them move the second blocks on the bottom up toward the others to show the joining of the two sets.
- 4. In subtraction have students place the first number on the work boards. The second number is then removed from the board. Sometimes the students choose to remove the tens or hundreds before the units. This action may be contrasted with how subtraction is performed from right to left with paper and pencil.
- 5. Use rubber stamps with base ten configurations on them to make materials for a learning center:
  - a. Pizza Cardboard Practice - divide a pizza cardboard into six to eight areas through the center. Stamp a different number representation onto each area. Laminate the cardboard and write the answer for each area on the reverse side with permanent marker.
  - b. Self Correcting Cards - use colored or white file cards. Stamp base ten configurations on one side. Write the standard numeral on the other. Laminate the cards.
- 6. Base Ten Activities (see below) has games which may be played using the base ten blocks. Instructions include the number of players, material needed and rules.
- 7. Have children build structures with the base ten blocks. They may estimate how much the blocks are worth and then check out their answer.

### **Materials Needed for the Lesson:**

The following materials may be purchased from: Creative Publications, P.O. Box 10328, Palo Alto, California 94303

1. Base Ten Activities: descriptions for teachers on how to teach free play, trading games, operations, counting, algebraic expressions, and spatial relationships.
2. Picturing Numeration: 194 pages of worksheets for students. It covers writing numerals, trading in addition, comparing numbers and joining amounts.
3. Picturing Addition: 188 pages of worksheets for students. It covers combining and recording, trading, and column addition up to four places.
4. Picturing Subtraction: 187 pages of worksheets for students. It covers giving away, trading, crossing out, and up to four place subtraction.
5. Picturing Multiplication and Division: 118 pages of worksheets for students. It covers repeated addition, and splitting or sharing of a total.
6. Base Ten Rubber Stamps: Excellent for making learning center materials or worksheets.

Powers of Ten for the Overhead Projector may be purchased from the Cuisenaire Company of America, Inc., 12 Church Street, Box D, New Rochelle, New York, 10805. These show the base ten blocks on transparancies

The Cuisenaire Company also has a base ten computer program for the Apple Computer. The base ten symbols are used to show the operations of addition, subtraction, multiplication and division. The program will be sent on approval for ten days at no cost. It is recommended that the program be tried before it is purchased.

### **Notes:**

**LESSON 3**  
**The Geoboard: Perimeter and Area**

**Teacher: Sally Cowger      Demonstration Grade: 5-6**

**Introduction to Geoboards:**

A geoboard is a square board with a lattice of pegs that children can use to study a variety of geometric concepts. It has been a popular manipulative aid since the 1960's. Primary students find it helpful in developing an understanding of geometric concepts and multiplication. For the intermediate students it gives hands-on experience with perimeter, area, coordinates and many fraction and decimal concepts. At all grade levels, the skills of listening and following directions can be reinforced when the geoboard is used.

Geoboards are a worthwhile addition to any elementary mathematics classroom. Since students are actively involved, they enjoy using the geoboard and generally motivation is not a problem. It is a good idea to have students working in pairs. In addition to the sharing which occurs, an extra hand is sometimes welcome.

Be careful not to rush the initial part of instruction. Students need time to get acquainted with the board informally, and to understand specific directions concerning how it will be used in the lesson being taught. This process takes time and patience.

**Background to the Lesson:**

In this lesson the concepts of perimeter and area are introduced to fifth and sixth grade students. For the purpose of demonstration they were presented more quickly than should actually be attempted. It is wise to move slowly from the concrete to the abstract as these concepts may be too difficult for the maturation level of many fifth grade students. At this age the initial exposure to the concepts is very important for future success.

**Questions to Guide Study of the Lesson:**

1. The students and teacher were relatively unsuccessful in establishing the practical applications of perimeter. What things could the teacher have done or said to accomplish this?
2. The teacher had all students respond and accepted all answers. Then, together the problem was solved. Do you feel this technique was effective? What benefits or drawbacks do you see with this approach?
3. What instances can you note from the lesson that showed that student discovery occurred using the geoboard?
4. How could you use the geoboard for concept building?

### **Further Suggestions:**

1. Create a geoboard learning center - activity cards are available on both primary and intermediate levels.
2. Have an 'all sports' math day (other themes can also be used). Have students find the area or perimeter of a gym floor, football field, wrestling mat, spectator's program, etc. Let your imagination go!
3. Students may use geoboards for coordinate games such as Battleships.
4. Students enjoy using dot paper and assigning letters to the dots and making their own puzzles.
5. Use geoboards to build other concepts, such as determining the area of triangles and other polygons, and graphing coordinates.

### **Materials Needed for the Lesson:**

The only materials needed for this lesson are plenty of rubberbands and geoboards. You may use one geoboard per pupil or have students share geoboards and work in small groups. Make sure you have lots of rubberbands on hand since they tend to break if stretched too far.

Geoboards, rubberbands and geoboard activity cards can be purchased commercially and are relatively inexpensive. Handmade geoboards, however, are just as effective and can provide an interesting project for a junior high or high school shop class. Geoboards can also be made by upper elementary age students (grades 3-6) or at home under parent supervision. This allows parents to participate in the project. The school can supply all materials or ask students to supply wood and nails. If the students supply materials they should be permitted to keep the geoboards at the end of the school year so they can enjoy them for many years to come.

To make geoboards students will need a square of plywood, nails with heads, and dot paper. Teachers can decide how big geoboards should be and how far apart nails should be located and then prepare dot paper the same size as the square of plywood and marked with dots where the nails will be pounded in to serve as pegs.

### **Notes:**

**LESSON 4**  
**The Geoboard: Attributes of Shapes**

**Teacher: Susan Stockdale      Demonstration Grade: 3-4**

**Introduction to Geoboards:**

The geoboard is a flexible aid for mathematics instruction. This lesson demonstrates another potential use. For more information also refer to lesson number 3.

Use of the geoboard increases the effectiveness of a lesson because it allows students to manipulate shapes. This adds kinesthetic experiences to the auditory and visual experiences typically used in geometry lessons.

**Background to the Lesson:**

The lesson begins with an introduction to the geoboard. Students then experiment with the geoboard and discuss what shapes can or cannot be made on a geoboard. In the last half of the lesson the class plays a game designed to help make students aware of the attributes of shapes.

This lesson was designed as an introductory lesson to the attributes of geometric shapes for third and fourth graders. It can also serve as an introduction to the geoboard. Third and fourth graders in this lesson have worked with geometric shapes, can label them, and are aware of shapes in their environment. This lesson extended student knowledge by making them aware of the various attributes of shapes, such as, the number and size of angles and the length of sides.

Challenging students is one of many motivational techniques. In this lesson a game format was used to challenge students and to provide focus and interest.

**Questions to Guide Study of the Lesson:**

1. In the lesson the teacher described one side of a shape by saying that it was four pegs long but only touched two pegs. What did she mean by this?
2. The students were confused by the above description. How could the teacher have eliminated this confusion?
3. Later in the lesson, the teacher tried to get students to describe the size of angles in their own words - for example, calling a right angle a square corner. Do you think this was important? The students did not want to give their own descriptions. Why not?

The teacher then supplied the description: "square corner, larger than square corner, smaller than square corner". Should she have done this? What else could she have done?

4. Why should students become aware of the attributes of shapes?

5. What events during the game showed that students needed more time to become familiar with the geoboard?

**Further Suggestions:**

1. Once students are familiar with a teacher's description, they can provide their own descriptions.
2. After students consistently make the shapes that are described, ask them to make their shapes in the same location as that of the describer. This activity can be used to lead into a lesson on the coordinate system.
3. Students can continue this activity (in pairs or small groups) in their free time.

**Materials Needed for the Lesson:**

The only materials needed for this lesson are plenty of rubberbands and geoboards. You may use one geoboard per pupil or have students share geoboards and work in small groups. Make sure you have lots of rubberbands on hand since they tend to break if stretched too far. The 'Materials for the Lesson' section of lesson three includes complete directions for making geoboards.

Companies selling geoboards and geoboard activity books or cards:

Creative Publications, 5005 W. 110th St., Oak Lawn, IL 60453

Cuisenaire Co. of America, Inc., 12 Church St., Box D, New Rochelle, New York 10802

Lakeshore Curriculum Materials Co., 2695 E. Dominguez St., P.O. Box 6261, Carson City, CA 90749

Activity Resources Company, Inc., Box 4875, Hayward, CA 94540  
Constructive Playthings, 1227 E. 119th St., Grandview, MO 64030

**Notes:**

**LESSON 5**  
**Tangrams for Teaching Geometry**

**Teacher: Sally Cowger      Demonstration Grade: 5-6**

**Introduction to Teaching Geometry Using Tangrams:**

The role of geometry in the elementary school curriculum is a topic of continuing discussion. Some teachers feel there isn't time to cover all mathematics areas, believe that geometry is a high school subject, or are simply uncomfortable with the topic.

Geometry should, however, receive a prominent place in the elementary curriculum because it helps to develop and reinforce students' spatial abilities. The ways we perceive our world are shaped by our experiences, culture and education. The traditional view has been that these spatial abilities are generally static and unchangeable. However, researchers are coming to believe that appropriate experiences can improve these perceptions.

Activities that introduce children to geometric concepts can encourage them to develop spatial abilities including: interpreting and making drawings, forming mental images, and visualizing movement and change. The use of manipulatives in mathematics instruction may also enhance the development of spatial perceptions.

An informal approach to geometry instruction, such as through use of puzzles, may fill the needs of both the student and the teacher. Puzzles can provide numerous worthwhile mathematical experiences for children and at the same time offer reluctant teachers a simple, yet exciting, means of introducing students to geometric concepts.

Puzzles can be used to teach geometry but confidence in puzzle solving must be developed through repeated student success. If a student is given a puzzle that is too easy, he/she will solve it and ask for a harder one. If the puzzle is too difficult, however, the student will quickly give up and lose confidence in his/her ability to solve puzzles. Puzzle skills can be enhanced by gradually increasing the difficulty of the task.

Puzzle solving can be used successfully throughout the elementary grades. It can help the kindergarten youngster learn about geometric shapes and properties. At the upper grades puzzles help teach congruence, similarity, area, and the Pythagorean Theorem. If your aim is to help students enjoy learning, develop creative, innovative thinkers and increase motivation, give geometry with puzzles a try. This can be a rewarding and worthwhile experience for all.

The tangram is a well-known geometric puzzle which was originated in China over 4,000 years ago. The object of the tangram is to assemble the seven pieces so as to form various figures. A great number of silhouettes of common objects can be formed. The tangram pieces are shown in Example 1 below.

### **Background to the Lesson:**

In the video tape, tangrams are introduced to the class through a paper folding activity in which students make their own tangrams. They are especially effective for teaching congruence and similarity and offer a concrete way to teach geometric shapes. Constructing a tangram reinforces the skills of listening and following directions. Students are highly motivated and become very quiet and absorbed in their task. The idea for this activity came from, Dorothy S. Russell and Elaine M. Bologna, "Teaching Geometry with Tangrams", Arithmetic Teacher, October 1982.

### **Questions to Guide Study of the Lesson:**

1. In this lesson, the students intensely concentrated on the task. If the teacher talks at times like this, what might the effect be?
2. Puzzle solving is an excellent experience in which to use cooperative learning. Can you think of other classroom seating arrangements than working in pairs?
3. One problem in puzzle solving is getting students to try. Can you think of ways to accomplish this?
4. Considering the time constraints placed upon classroom instruction, what are the pros and cons of teaching geometry using puzzles?
5. Can the use of puzzles reduce math anxiety? How?
6. Making original puzzles is a confidence builder and results in enjoyable learning experiences. Share ideas and insights you have gained from this lesson which you can implement in your classroom.

### **Further Suggestions:**

1. Have a puzzle math learning center. Be sure to include original student puzzles.
2. Give puzzle figures an appropriate background and use them to decorate the bulletin board or classroom walls.
3. Use puzzle shapes to create a pattern, make a border design, or make a picture.
4. Use puzzle pieces to introduce students to tessellations. These can be an extension or reinforcement of the geometric concepts already taught or may lead to other more advanced concept development.
5. Invite students to bring in challenging puzzles for the class to share.
6. Other geometric concepts to introduce using puzzles include: slides, turns, flips (mirror image); area of other polygons; and working with angles.

#### **Materials Needed for the Lesson:**

Example 1, that follows this lesson, presents activities to guide construction of tangrams through paper folding and mathematical problem solving.

You and your students may want to create puzzles for your classroom using patterns and ideas from this lesson. Puzzles can be made from a variety of materials including balsa wood, masonite, vinyl tile, tagboard, colored plastic, and heavy paper. Lamination will give the paper version a longer classroom life.

Every effort should be made to insure precision in making puzzle pieces. Poke pins through the vertices of the pattern onto the puzzle materials, connect with a straightedge, and cut with a sharp pair of scissors or an Xacto knife. The puzzle is more likely to be precise if it is copied from the completed puzzle pattern rather than each piece copied separately. A word of caution - there are two ways to give away the position of a puzzle piece. These are:

1. Using a grained paper or material that makes it obvious that all pieces must fit in such a way that the grain is parallel.
2. A poorly made cut on an edge between two pieces may make it obvious that those two edges are made to fit together.

Puzzle patterns can be found in Pic-A-Puzzle, A Book of Geometric Puzzle Patterns, by Reuben A. Shadler and Dale G. Seymour. The Cuisenaire Company Catalog, and Tangram (a game by Discovery Toys) are other useful resources.

#### **Notes:**

## Example 1

### Constructing Tangrams through Paper Folding

Materials: sheet of paper (8 1/2" by 11"), pencil, scissors.

(A set of the seven tangram pieces can be constructed from a transparency and then used on the overhead projector to demonstrate the various steps as you progress.)

1. Fold the piece of paper to make a square by bringing point *a* to point *b* and creasing the paper (fig. 1). Cut off the surplus. Discuss the shape you now have.

2. Now fold the paper again, along the other diagonal. Discuss the shapes (triangles) made by these folds. Cut along one diagonal to form two large triangles (fig. 2). Have children discuss the size of these two triangles.

3. Cut one large triangle along the center line to form two smaller triangles (fig. 3). Label them 1 and 2. Students can be asked questions like the following ones. (Related mathematical ideas are in parentheses.)

- Are shapes 1 and 2 alike? How? (Congruency)
- What do you notice about the corners of these triangles? (Size)
- Are any of the triangles' corners square? (Right angle)
- How can you prove corners are square? (Put the corner against the corners of floor tiles, a desk, cover of a book, room, and so on. Tell children that a triangle that has one square corner or right angle is called a right triangle.)
- Are there any other square corners in the room?
- Are all square corners congruent? (Have children prove this by putting a right angle up to other square corners or right angles. Introduce the measure of a right angle as 90°.)
- How would you describe the sides of the right triangle? Are any sides longer than the others? (Introduce *hypotenuse*, the name of the side opposite the right angle.)
- Look at the remaining triangle. Does it have a right angle? Where is its hypotenuse?

Fig. 1

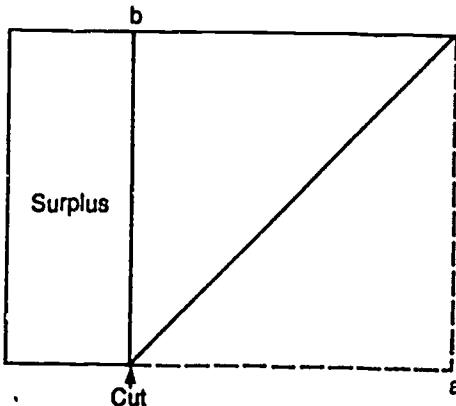


Fig. 2

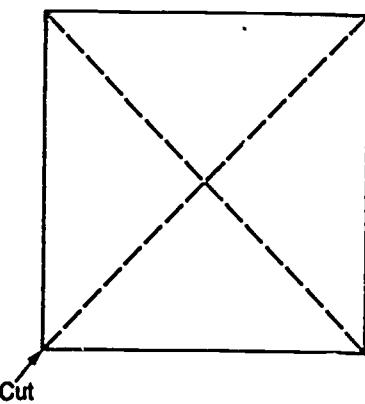


Fig. 3

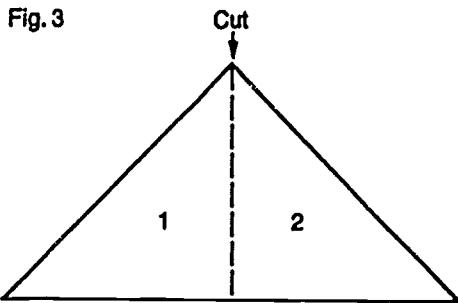
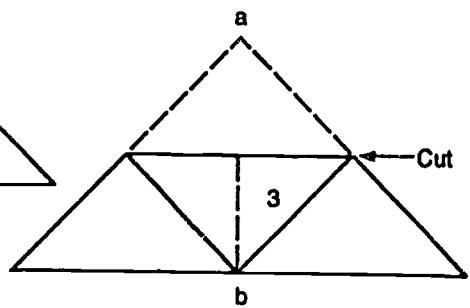


Fig. 4



4. Holding the remaining large triangle, fold the corner or the right angle to the midpoint of the hypotenuse and crease (fig. 4). Cut along the crease and label the new triangle 3.

2. Each angle of one triangle is congruent to the corresponding angle of another of the right triangles.)

- Is shape 3 congruent to shapes 1 and 2? Does 3 have a right angle? Is it a right triangle? Where is the hypotenuse of triangle 3? What do you notice about the sizes of the other angles of the right triangles? (Shape 3 is similar to shapes 1 and
- Ask students to describe the remaining piece. (A trapezoid is a four-sided figure with two parallel and two nonparallel sides.)
- Does the trapezoid contain any right angles? Are any of its angles congruent to any other angles? How do you know this? (Place smaller angle of right triangle over smaller angle of trapezoid; place

#### Questions for students:

- Is shape 3 congruent to shapes 1 and 2? Does 3 have a right angle? Is it a right triangle? Where is the hypotenuse of triangle 3? What do you notice about the sizes of the other angles of the right triangles? (Shape 3 is similar to shapes 1 and

Fig. 5

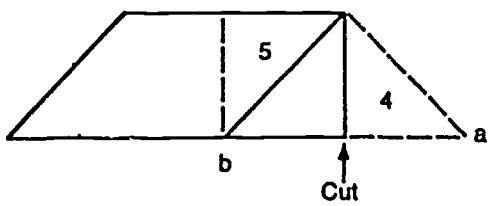


Fig. 6

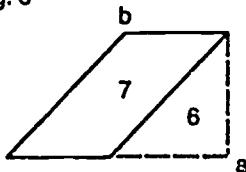


Fig. 7

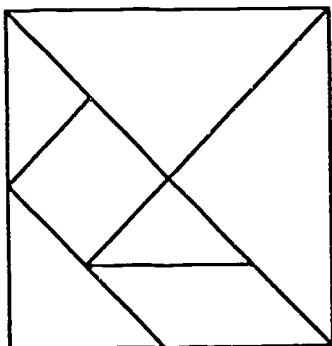


Fig. 8

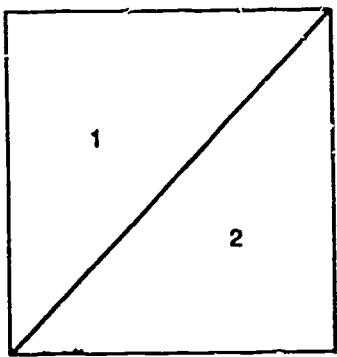


Fig. 9

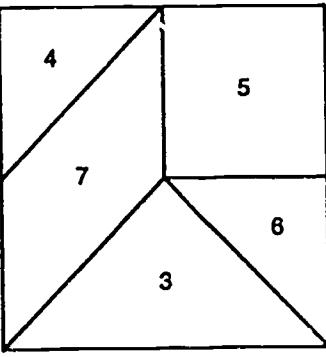


Fig. 10

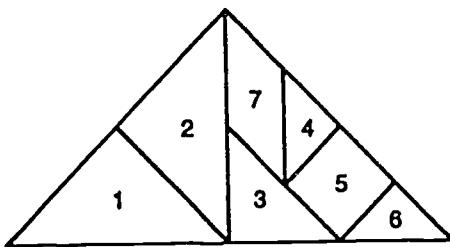
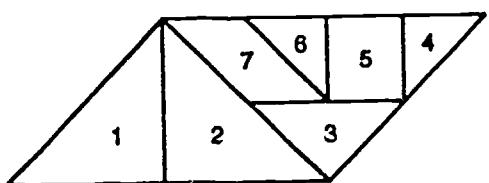


Fig. 11



right angle plus smaller angle over largest angle of trapezoid.)

5. Fold one corner, *a*, of trapezoid to midpoint of baseline, *b* (fig. 5). Crease. Cut along crease and label this triangle 4. Cut off remaining square and label it 5.

Questions for students:

- Is 4 congruent to any other triangles? Does it contain a right angle? What kind of a triangle is it? Where is its hypotenuse?
- Looking at 1, 3, and 4 how are they alike? How are they different? (Similar)

6. Fold the remaining piece so that you obtain another right triangle and a parallelogram (fig. 6). Bring point *a* to point *b* and crease. Cut and label the triangle 6 and the parallelogram 7.

Questions for students:

- Is shape 6 congruent to any other piece? Is it similar to another piece? What do you know about the angles in 6?
- How would you describe shape 7? (A parallelogram is a four-sided figure and the opposite sides are parallel.)
- Does 7 contain any right angles? Are any angles of 7 congruent to any other angles of the triangles? (Help children discover that the smaller angles of the parallelogram are congruent to the smaller angles of the right triangles. Also, the larger angle of the parallelogram is congruent to a right angle plus the smaller angle of the triangle.)

Reproduced with permission  
from Russell, Dorothy S.  
and Bologna, Elaine M.,  
"Teaching Geometry with  
Tangrams," Arithmetic  
Teacher, Oct. 1982, pp.  
34-38.

**LESSON 6**  
**Extending the Mathematics Worksheet**

**Teacher: Francie Turner      Demonstration Grade: 2-3**

**Introduction to Extending Worksheets:**

A mathematics worksheet may be used to practice whatever the directions dictate. The same sheet can then be used to study vocabulary, concepts, or other mathematical ideas that need to be reinforced. This process of intensifying and extending a worksheet is an adaptation of Dr. John Manning's technique with reading worksheets. The purpose of intensifying and extending is two-fold: 1) to use the worksheet to practice more than its original content, and 2) to actively engage students so that they spend more time on task and increase the probability that learning will take place.

Advantages of this procedure include: 1) requiring that fewer worksheets be completed by students; 2) being better able to see if a student is having difficulty, and intervene before anxiety sets in; 3) helping students learn to mentally 'switch gears' from one mathematical idea to another in a short period of time; and 4) encouraging students to actively participate in the learning activity by indicating an answer or interacting in a discussion.

**Background to the Lesson:**

Before the videotape lesson begins, students have completed and checked a money worksheet. In the taped lesson the teacher uses that worksheet to reinforce and introduce various mathematical concepts.

**The following procedures guide use of this strategy:**

1. The teacher should have an exact copy of the student's worksheet. (When working with a small group the teacher may have a paper sheet and sit at the long side of the table so all students around the table can see, or the teacher may make an overhead transparency of the worksheet.) Upon giving directions the teacher illustrates how to do it so the students have a copy to imitate.
2. Besides numbering problems, students can be directed to draw lines to make rows of problems.
3. To encourage students to pay attention do not review problems consecutively, instead move randomly around the page, e.g., problem 15, problem 2, problem 6.
4. Students should be actively writing so that as many senses as possible (visual, motor, tactile) are used.

**Questions to Guide Study of the Lesson:**

1. How does the teacher choose which terms or concepts will be practiced on the worksheet?

2. This process of extending need not be limited to money worksheets. Name two other worksheets where this process could be used.
3. Each grade level has certain mathematical concepts that need reinforcement. Make a list of concepts taught at your grade level. Which could be reinforced using an addition or subtraction worksheet you normally give your students?

**Further Suggestions:**

1. Worksheets from other subject areas, such as science and social studies, may also be used to extend mathematics concepts.
2. Worksheets in other subject areas can also be used to extend concepts within these subject areas. For example, a reading worksheet may be used to reinforce other reading concepts than those for which it was originally designed.

**Notes:**

**LESSON 7**  
**Probability: An Introduction**

**Teacher: Susan Stockdale      Demonstration Grade: 3-4**

**Introduction to Probability:**

Many of today's mathematics textbooks include lessons on probability. Therefore, it is important to understand this concept and have some idea of how to go about teaching it. But what is probability?

Probability allows us to quantify the chances of some event occurring. It allows us to make educated predictions and formulate hypotheses.

Because the exact mathematical formulas are complex, students at the elementary level should not be required to quantify every probability experience. The main concern at this level should be to provide many different experiences that will help students begin to develop or further develop intuitive knowledge of probability.

It is important to limit the first discussions in an introductory lesson to one attribute rather than several attributes or combinations of attributes. This avoids confusion and needless frustration. Once students become competent at finding probabilities for one attribute the teacher can then present combinations of attributes and show how probability changes when one member of a set has been drawn. When learning about probability it is also very important that students receive hands-on experience. This makes probability more real and meaningful to students and thus easier to learn.

**Background to the Lesson:**

In this lesson, students learn to quantify probability using the form  $X$  in  $Y$ , (i.e., 1 in 12). It was decided not to teach the form  $X/Y$ , (i.e., 1/12) because of its similarity to the symbolic representation of fractions. Third and fourth graders are just learning about fractions and learning the  $X/Y$  notation for probability may interfere with previous or subsequent learning about fractions.

Attribute blocks and a deck of cards were used in this lesson because both can vary in more than one way. Attribute blocks vary by size, shape, color, and thickness. Cards vary by suit and number. This allows for discussion of the probability of drawing objects by an attribute or combination of attributes. For example, the probability of drawing blocks by color (red, yellow, blue) or by color and shape, (i.e., a yellow square) may be discussed.

In this lesson the initial discussion focuses on the probability of drawing a specific colored block at random. Later, combinations and changes in probabilities were studied. Another group of students may take longer to cover these ideas. That is to be expected. Let students set the pace of instruction.

### **Questions to Guide Study of the Lesson:**

1. What indications are there that some students may already have an intuitive grasp of probability?
2. At several points the teacher used the terms 'odds' and 'probabilities' interchangeably. Are they interchangeable? Explain.
3. Probability can be taught using any group of objects that vary in at least one way. Attribute blocks and a deck of cards were chosen for this lesson. What other materials could be used?

### **Further Suggestions:**

1. Have students discuss how knowledge of probability can help them. For example, when is probability used or when can it be used by students? (You could have them bring in examples from newspapers or magazines.)
2. The three topics discussed in this lesson: determining probability, what happens when one member of a set is drawn, and whether one attribute or a combination of attributes is easier to predict, can be explored further.

You can move to more advanced topics if you and your students are confident and enjoying probability. For example, study the difference between determining probabilities and determining odds.

To decide what additional topics can be explored you will want to broaden your knowledge of probability and then determine what your students should know and/or what your students can handle. Most statistic books and textbooks on elementary mathematics methods for teachers include helpful chapters on probability. If none are available, you might ask a high school or junior high mathematics teacher to help or to recommend a good book.

3. The following list of materials and activities is presented to stimulate your creative thinking of ways to present probability.
  - a. Dice - Students roll two dice and keep track of the sum or product of the numbers on the dice. Have students predict the most frequent product or sum. Students might also predict how many times they would roll the dice before each number, 1-6, has been rolled once.
  - b. Mystery bag. The mystery bag is a small cloth or paper bag into which a variety of objects can be placed. The following is a description of a simple experiment which illustrates one way to use the mystery bag:

Place marbles of two colors in the bag. Ask students to draw a marble, record its color, and then replace it in the bag. After this procedure has been performed many times (20-50), have students state what colors are in the

bag and which color represents the most or least number of marbles. Students might also state the probability of drawing each color. After students have stated their beliefs, have them open the bag and verify their guesses.

- c. Spinner activities. Partition spinners into halves and into two regions of unequal area. Tell students they will be assigned a color and given a point each time the spinner stops on their color. Discuss which color they will want and why. Spinners may also be divided into more than two regions for a variation of this activity.
- d. Basketball simulation. This activity simulates the one-and-one foul shot in basketball. The results of this situation in real life may be 0, 1, or 2 points. Ask students to suppose a player has a 60% free throw percentage. Have students predict which outcome (0, 1, or 2 points) occurs most often when that player goes to the free throw line. Check predictions by constructing a spinner on which 60 percent is marked to show a basket and 40 percent represents a miss. Have students experiment and compare results with predictions. Repeat this experiment using different shooting percentages. Compare and discuss results.
- e. Probability experiments. Let your students determine the probability of something such as a flipped coin landing with heads or tails up. Let students repeat this experiment many times making predictions and checking reality against the predictions. For example, students might predict how many heads they will get if they flip a coin 50 times and check results against their prediction. Students can keep track of results by making graphs or tables in order to reinforce graph and table making skills.

**Notes:**

Lesson 8  
Teaching Problem Solving Skills

Teacher: William Tomhave      Demonstration grade: 5-6

**Introduction to Problem Solving:**

Much has been written in recent years about the need for schools to teach students how to use higher order thinking skills. One of these skills, problem solving, has long been considered to be within the domain of mathematics instruction. Unfortunately, the interpretation given to 'problem solving' has been frequently limited to solving the typical word problems that are direct applications of computational skills.

Krulik and Rudnick offer the following definition of problem solving: "Problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation." (pg 4). By the early 1980's, several organizations and individuals had produced materials designed to help teachers teach problem solving. The materials carefully distinguished between teaching problem solving and doing word problems. Much of the work built on the base of George Polya's, How to Solve It, originally published in 1945.

**Background to the Lesson:**

In this lesson the teacher uses the problem - how many squares are on a checker board - to introduce the problem solving techniques of: solve a simpler problem, make a table, and look for a pattern.

The approach to problem solving that this lesson demonstrates is described in Problem Solving: A Basic Mathematics Goal in which it is noted that a mathematical problem must possess the following characteristics:

- A situation must exist involving an initial state and a goal state.
- The situation must involve mathematics.
- A person must desire a solution (must accept the problem).
- There must be some blockage between the given and desired states.

(Ohio State Department of Education, pg 5)

**Questions to guide study of the lesson:**

1. The lesson introduces students to several problem solving approaches. List the strategies used, and briefly describe what each strategy entails.
2. For each strategy you identified in (1) above, try to locate a problem that you could use to demonstrate it.

3. The particular problem discussed in this lesson generated the series  $1 + 4 + 9 + \dots + 64$ . What are the terms of this series called? Can you think of another situation in which this particular sequence occurs?
4. One important consideration in teaching problem solving is the need for students to believe that they can solve the problem. How might calculators be used to advantage?
5. Teachers generally like to talk. How could teacher talk be more of a hindrance than a help for students who are trying to solve a problem?
6. When one student sees a pattern, it is easy for the teacher to assume that both that student and the classmates understand the problem. Is this always true? What could a teacher do to verify that the students did in fact understand the problem?
7. Some people have acknowledged that problem solving is important, but believe that it is not reasonable to include it as part of the primary curriculum. How do you react to this position?
8. Time is a key issue in teaching problem solving. It is unlikely that any more time will be added to the school day to make room for teaching this skill. What might be given less emphasis in your curriculum to make room for the teaching of problem solving?
9. How would you explain to another teacher that the word problems in a typical elementary textbook are really not problem solving?

#### **Further Suggestions:**

The study of problem solving can be promoted through ideas such as the following: (1) set up a problem solvers' learning center; (2) have a 'problem of the week' with an 'honor roll'; (3) set aside a regular time each week for non-routine problems; and (4) invite students to bring in challenge problems for the class.

#### **Materials Needed for the Lesson:**

A variety of materials are available for educators interested in teaching problem solving. These include: Techniques of Problem Solving (TOPS) materials by Immerzeel et. al.; Problem Solving Books I and II by the Ohio State Department of Education; and The Problem Solving Handbook and Resource Book by Krulik and Rudnik. (See bibliography for complete references).

Techniques of Problem Solving, Greenes, Immerzeel, Ockenga, Schulman, and Spungin. Available from Creative Publications, P.O. Box 10328, Palo Alto, CA 94303 and Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303. Card decks (A, B, C, D, AA, BB, CC, DD, AAA) are also available for many grade levels along with problem solving skill sheets and developmental worksheets.

Problem Solving Through Strategy Games, Fred Nolan, Olivia Public Schools, Olivia, MN 56277. A series of computer problems that teach a strategy game and with it one or more problem solving heuristic.

Notes:

**LESSON 9**  
**Base-Ten Blocks: Teaching Decimals**

**Teacher: Sally Cowger      Demonstration Grade: 5-6**

**Introduction to Decimals:**

Decimals have always been important. With increased use of calculators and the metric system they must receive an even more prominent place in the elementary mathematics curriculum.

Although teachers often think of decimals as easy to use in computation, students frequently do not grasp the conceptual meaning of a decimal. A study that looked at the initial learning of decimal concepts in grades four through six concluded that teachers should spend more time having children become familiar with decimals - constructing models, saying them, counting them, writing them, comparing them with their equivalent fractions - before rushing to decimal computation. The study suggested that the decimal concept might be too difficult for many fourth and fifth grade students. While exposure to the concept of decimals at these levels is crucial, the results in grade six were much more positive.

**Background to the Lesson:**

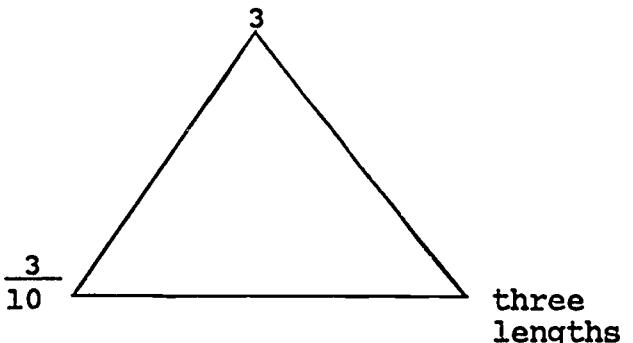
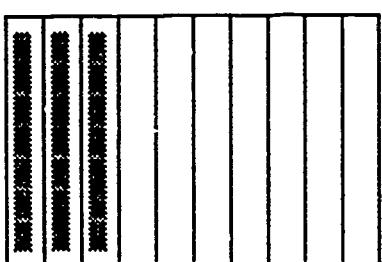
Base ten models are excellent manipulatives to use at all grade levels for teaching a variety of mathematical concepts. They are as effective in teaching place value in decimals as they are for teaching place value using whole numbers in earlier grades. One problem in the use of base ten models is the cost of commercially produced sets. Inexpensive base ten sets can be made from laminated centimeter graph paper. A math supply cupboard where materials can be checked out and shared by all elementary teachers as they are needed for instruction is another, and inexpensive, way to have manipulatives available for mathematics instruction.

In the lesson, fifth and sixth grade students are being asked, "What is a decimal?" Students explore the answer to this question through brainstorming, concrete models (straws and a candy bar), and the base ten blocks.

**Questions to Guide Study of the Lesson:**

1. In this lesson students are brainstorming. The teacher accepted all answers even though some were inappropriate. What are the pros and cons of this approach?
2. Manipulation of base ten models takes valuable mathematics instruction time. Can you identify potential benefits derived from using these materials?
3. Do you think using manipulative materials and concrete examples can generally lead to student discovery? If not, what else could we use to facilitate this?
4. How is verbalizing mathematics related to understanding a mathematics concept?

5. Besides the examples below, what are some semi-concrete ways that students could move closer to the abstract idea of a decimal prior to undertaking pencil and paper tasks?



**Further Suggestions:**

When working with decimals, many materials are available to enhance instruction. In addition to concrete examples, base ten models and triangular models (semi-concrete), consider these possibilities:

Numberlines

Catalogs

Coupons

Newspapers

Advertising--

grocery stores

sporting goods catalogs

hardware stores

toy catalogs

drug stores

others--students' choice

Menus from restaurants

Play money

Real change

Games, such as Monopoly

Measurement activities

**Materials Needed for the Lesson:**

Base Ten Blocks - Available from Cuisenaire Company of America, Inc., 12 Church Street, Box D, New Rochelle, New York, 10805.

Graphic models using base ten models in a triangular format as suggested by Judith Zawojewski in "Initial Decimal Concepts: Are They Really So Easy?" Arithmetic Teacher, March 1983, pg 52-56.

**Notes:**

**LESSON 10**  
**Introduction to Attribute Blocks**

**Teacher:** Mary Lou Smith      **Demonstration grade:** K

**Introduction to Attribute Blocks:**

Attribute Blocks are used as an effective teaching aid for such skills as distinguishing attributes and carrying out logical operations. The blocks possess several characteristics (size, color, shape, and thickness) that may be used to discuss classifying, ordering, and other skills of logical thinking. As in the case of other manipulatives, it is important that children have time to familiarize themselves with the attribute blocks. If adequate time is not provided, children will likely ignore a regular lesson and use the time to play with the blocks. Once children have become familiar with the blocks and their uses, several concepts can be taught:

- sorting by different attributes,
- looking for differences,
- ordering according to size,
- learning correct terminology,
- learning about sets and set operations.

Experience has shown that children working with these blocks are motivated to stay on task for a long period of time. Learning takes place because the manipulatives provide a concrete representation for abstract ideas and because children are actively involved and interacting with their peers.

**Background to the Lesson:**

This lesson begins with the teacher discussing the concept of attribute, and moves to review and reinforce the ideas of basic shapes. After being introduced to the materials, students make shapes on the chalkboard. As they draw the shapes the work is discussed and proper terminology is used. The teacher observes individual differences as the children are actively involved with the pattern blocks.

Next children work in pairs to complete a task as the teacher moves about the classroom to offer suggestions, supportive comments, and observations about each child's work. This approach requires fewer materials, and allows children to correct each other's mistakes without teacher intervention. Finally, the teacher leads from a general discussion of attributes to set concepts.

**Questions to Guide Study of the Lesson:**

1. Some children will sit and not get involved with a lesson. How might the teacher work to get these students involved? What is the best solution? Can you suggest other options?
2. Sometimes, it is helpful to show objects in the real world that possess the same characteristics as the objects being used in a lesson. Find examples of objects in the world around you that have the same shape as the blocks.

3. What major characteristics and other uses can you list for attribute blocks?

**Further Suggestions:**

As children become familiar with attribute blocks other mathematics topics such as set operations can be introduced. The blocks are a natural resource for teaching set unions and intersections. They can also help students look for patterns and to form logical inferences. This is done by having students observe a sequence of blocks that differ by one or more attributes and then determine the next piece in a series.

**Materials Needed for the Lesson:**

Attribute Blocks are available from Creative Publications, 5005 W. 110th St., Oak Lawn, IL 60453 and from Cuisenaire Company of America, Inc., 12 Church Street, Box D, New Rochelle, New York, 10805.

**Notes:**

**LESSON 11**  
**Introducing Pattern Blocks**

**Teacher: Mary Lou Smith      Demonstration Grade: K**

**Introduction to Pattern Blocks:**

Pattern blocks are a collection of brightly colored geometric shapes that provide a versatile manipulative for mathematics instruction. The blocks may be used to copy shapes, make patterns, display logical thinking and problem solving, and to investigate geometric concepts such as symmetry, area, perimeter, and congruence.

In order to use the blocks successfully, children must have the opportunity to use them in free play. During this exploring time they should become familiar with the pieces, explore relationships between the pieces, and make original designs. The amount of time needed for this exploration will vary greatly from child to child, but it is a crucial starting point for an effective lesson. As children become familiar with the blocks, it is important to discuss the correct names of the pieces so that the pieces can be identified by both name and color in later activities.

**Background to the Lesson:**

In this pattern block lesson, Kindergarten children are introduced to the blocks. They engage in a range of activities including learning terms and making designs.

**Questions to Guide Study of the Lesson:**

1. In order to help students see how they could cover one shape with other shapes, the teacher was very directive. What questions would you use to lead the children to discover the desired relationships on their own?
2. When the children are involved with worksheets they readily stay on task and share their discoveries with others. One student asked, "Do they get any harder?" What was his response to the teachers affirmative answer? Is this the way your students react when they know more difficult material is forthcoming?
3. To carry out many of the patterning activities it is necessary to be able to exchange objects for an object of the same shape. What object can be made from two red trapezoids? Find other combinations of blocks that will cover another block exactly. List your findings.
4. What are the names of the pattern block pieces? Are you familiar with these terms? If you are unsure of the meaning or pronunciation of a term, where would you go for help?
5. For each of the basic block shapes in (4) above, can you identify something in your everyday world that possess the same basic shape?

#### **Further Suggestions:**

Pattern blocks provide an enjoyable mathematics experience that leads to enthusiastic class interaction, a high level of motivation, and experience with many geometric concepts. To reinforce use of proper terminology, make templates of the basic shapes and display them on the chalkboard or bulletin board. As children experiment and discover new patterns, encourage their use in a related art project.

Commercially available worksheet activities also provide practice in problem solving and logical thinking. Other suggestions for using pattern blocks include:

- Copy a design.
- Fill a region with the most/fewest blocks possible.
- Reproduce a design after it is reflected in a mirror.
- Extend a pattern to fill a place region.
- Extend the notion of patterning to other tessellations.
- Display the work of artist M. C. Escher and make comparisons.

#### **Materials Needed for the Lesson:**

Attribute Blocks, are available from Creative Publications, and from Cuisenaire Company of America, Inc., 12 Church Street, Box D, New Rochelle, New York, 10805.

#### **Notes:**

**LESSON 12**  
**Attributes of Shapes Using Pattern Blocks**

**Teacher: Mary Lou Smith      Demonstration Grade: K-1**

**Introduction to Pattern Blocks:**

Shapes and patterns seem to be especially fascinating to young children. They play with them and make discoveries spontaneously. Even first graders can learn to use proper terminology and find relationships between shapes. It is important to note that children need experimental time. Children will play with blocks, and through this play will discover many relationships among the pieces on their own. If children are not given enough time to handle the materials, they will ignore the teacher until they have satisfied their curiosity.

**Background to the Lesson:**

The lesson begins with a review of geometric shapes. The teacher then allows students to experiment with the blocks and aids student discovery. Finally, students use the blocks to fill in outlines on a worksheet.

Too much teacher talk, or questions that fail to give adequate response time, only detract from the lesson. This particular lesson points out the problems of trying to teach too many new concepts in too short a time.

**Questions to Guide Study of the Lesson:**

1. Students need time to become familiar with the blocks. When the teacher tried to move them away from free play to a worksheet, she was met by passive resistance. What does this reveal?
2. The students were interested in sharing their findings with others. How might the teacher take advantage of this? What problems can result?
3. When given the opportunity to build things the children made many interesting structures. How was this building related to the lesson objective of learning about the relationship between the various shapes?
4. When the students were busy with their worksheets, what happened to the noise level in the classroom and to their activity level? Is a relatively high noise level necessarily related to a high level of activity?
5. In this lesson, students were encouraged to use proper terminology when discussing particular pieces. Do you agree with this approach? What is the most effective way to do this?

**Further Suggestions:**

See further suggestions from Lesson 11.

**Materials Needed for the Lesson:**

Attribute Blocks, are available from Creative Publications, and from Cuisenaire Company of America, Inc., 12 Church Street, Box D, New Rochelle, New York, 10805.

**Notes:**

## BIBLIOGRAPHY

Bennett, Albert B., Jr., and Nelson, Leonard T., Mathematics - An Activity Approach, Allyn & Bacon, Inc., 1979.

Davidson, Patricia and Robert Willcutt, Spacial Problem Solving with Cuisenaire Rods, Cuisenaire Corporation, 1983.

Fey, James T., "Mathematics Teaching Today: Perspectives from Three National Surveys for the Elementary Grades" in What Are the Needs in Precollege Science, Mathematics, and Social Science Education? Views From the Field, National Science Foundation, Office of Program Integration, Directorate for Science Education, SE 80-9.

Gardner, Martin, Aha! Gotcha, W. H. Freeman, 1982.

Gardner, Martin, Aha! Insight, W. H. Freeman, 1982.

Gibney, Thomas and Karns, Edward, "Mathematics Education 1955 - 1976" in, What Are the Needs in Precollege Science, Mathematics, and Social Science Education? Views From the Field, National Science Foundation, Office of Program Integration, Directorate for Science Education, SE 80-9.

Harvey, Linda and Ann Roper, Pattern Blocks Problems for Primary People, Creative Publications, 1979.

Jackson, Allison L., "Mathematics Education" in, What Are the Needs in Precollege Science, Mathematics, and Social Science Education? Views From the Field, National Science Foundation, Office of Program Integration, Directorate for Science Education, SE 80-9.

Jacobson, Marilyn Hall, "Teaching Rational Numbers--Intermediate Grades," Arithmetic Teacher, February 1984, pp.40-42.

Krulik, Stephen and Jesse A. Rudnick, Problem Solving A Handbook for Teachers, Allyn and Bacon, Inc., 1980.

Krulik, Stephen and Jesse A. Rudnick, Problem Solving A Resource Book for Teachers, Allyn and Bacon, Inc., 1984.

Lloyd F. Scott as quoted in Mathematics an Activity Approach by Albert B. Bennett, Jr., and Leonard T. Nelson Allyn and Bacon, Inc., 1979.

Melton, Raymond, G., "Summary of the Findings of Three NSF Sponsored Studies on the Status of Precollege Science Education" in, What Are the Needs in Precollege Science, Mathematics, and Social Science Education? Views From the Field, National Science Foundation, Office of Program Integration, Directorate for Science Education, SE 80-9.

Nelson, Rebecca S. and Whitaker, Donald R., "Another Use of Geoboards", Arithmetic Teacher, April 1983, pp. 34-38.

Ohio Department of Education, Problem Solving: A Basic Mathematics Goal, Books 1 and 2, Ohio Department of Education, Columbus, Ohio.

Solja, George, How to Solve It, Princeton University Press, 1945.

Pasternak, Marian and Linda Silvey, Pattern Blocks Activities Books A and B, Creative Publications, 1975.

Russell, Dorothy S. and Bologna, Elaine M., "Teaching Geometry with Tangrams", Arithmetic Teacher, Oct. 1982, pp. 34-38.

Schadler, Reuben A. and Seymour, Dale G., Pic-A-Puzzle, Creative Publications, Inc., P. O. Box 328, Palo Alto, California 94302; 1970.

Schulte, Albert P. and Choate, Stuart A., What Are My Chances, Creative Publications, Inc., P. O. Box 328, Palo Alto, California 94302; 1970.

Spitler, Gail, "The Shear Joy of Area", Arithmetic Teacher, April 1982, pp. 36-38.

Young, Jerry L., "Improving Spatial Abilities with Geometric Activities", Arithmetic Teacher, September 1987, pp. 38-43.

Zawojewski, Judith, "Initial Decimal Concepts: Are They Really So Easy?", Arithmetic Teacher, March 1983, pp. 52-56.